

# Spin-3/2 pentaquark in the QCD sum rule

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## Abstract

We study  $IJ^P = 0\frac{3}{2}^\pm$  and  $1\frac{3}{2}^\pm$  pentaquark states with  $S = +1$  in the QCD sum rule approach. The QCD sum rule for positive parity states and that for negative parity are independently derived. The sum rule suggests that there exist the  $0\frac{3}{2}^-$  and the  $1\frac{3}{2}^-$  states. These states may be observed as extremely narrow peaks since they can be much below the  $S$ -wave threshold and since the only allowed decay channels are  $NK$  in  $D$ -wave, whose centrifugal barriers are so large that the widths are strongly suppressed. The  $0\frac{3}{2}^-$  state may be assigned to the observed  $\Theta^+(1540)$  and the  $1\frac{3}{2}^-$  state can be a candidate for  $\Theta^{++}$ .

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## I. INTRODUCTION

Recent observation of an exotic baryon state with positive strangeness,  $\Theta^+(1540)$ , by LEPS collaboration in Spring-8 [1] and subsequent experiments [2, 3, 4, 5, 6, 7, 8, 9, 10] has raised great interests in hadron physics. This state cannot be an ordinary three-quark baryon since having positive strangeness, and the minimal quark content is  $(uudd\bar{s})$ . A remarkable feature of  $\Theta^+(1540)$  is that the width is unusually small ( $\Gamma < 25$  MeV) despite the fact that it lies about 100 MeV above the  $NK$  threshold. The absence of isospin partners suggests that the  $\Theta^+$  is an isosinglet [3, 5]. The spin and parity have not yet been experimentally determined.

The discovery of  $\Theta^+$  has triggered intense theoretical studies to understand the structure of the  $\Theta^+$  [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. One of the main issues is to clarify the quantum numbers, especially, the spin and the parity, which are key properties for understanding the abnormally small width.

Following a naive expectation from ordinary hadron spectra, it is natural to assume that the  $\Theta^+$  has the lowest spin  $J = \frac{1}{2}$ . In fact, most of the existing works on  $\Theta^+$  in lattice QCD [13, 14] or in QCD sum rule [15, 16, 17, 18] have focused only on the  $J = \frac{1}{2}$  states. However, we cannot exclude the possibility that  $\Theta^+$  is a higher spin state, as suggested in some literatures [19, 20, 21, 22, 23]. For example, negative parity  $J = \frac{3}{2}$  states are especially important because they can be extremely narrow in the following reasons: First, consider an  $IJ^P = 0\frac{3}{2}^-$  state ( $I$  and  $P$  denote the total isospin and parity, respectively). If this state lies much below the  $NK^*$  threshold, no  $S$ -wave decay channel opens and the decay is restricted only to  $D$ -wave  $NK$  states. Due to the high centrifugal barrier the width is strongly suppressed. Thus, this state can be a candidate for the observed  $\Theta^+$ . For just the same reason,  $1\frac{3}{2}^-$  can also be seen as a narrow peak. If the state is sufficiently below the  $\Delta K$  threshold, the allowed decay channel is only  $NK$  in  $D$ -wave and the width can be significantly small [19, 24].

Another state in which we have an interest is the pentaquark with  $IJ^P = 0\frac{3}{2}^+$ , which has been discussed as a spin-orbit ( $LS$ ) partner of the  $0\frac{1}{2}^+$  state [22]. The  $0\frac{1}{2}^+$  state was assigned to the observed  $\Theta^+$  by Jaffe and Wilczek [11]. According to their conjecture, the  $\Theta^+$  consists of two spinless  $ud$ -diquarks in  $P$ -wave and an anti-strange quark. As a result, the  $LS$ -partners,  $0\frac{1}{2}^+$  and  $0\frac{3}{2}^+$ , appear due to the coupling between the relative motion with

orbital angular momentum  $L = 1$  and the intrinsic spin of the anti-strange quark. The effect of the  $LS$  force is so weak in the diquark structure that the mass splitting should be small. It leads to a possibility of the low-lying  $\Theta^*(IJ^P = 0\frac{3}{2}^+)$  as pointed out in Ref.[22].

In this paper, we study the  $0\frac{3}{2}^\pm$  and  $1\frac{3}{2}^\pm$  pentaquark states by using the method of QCD sum rule [25]. In order to ascertain the existence of the  $0\frac{3}{2}^-$  and  $1\frac{3}{2}^-$  narrow pentaquarks, it is crucial to estimate their absolute masses since their widths are sensitive to the energy difference from the  $NK^*$  or  $\Delta K$  threshold. QCD sum rule is closely related to the fundamental theory and is able to evaluate the absolute masses of hadrons without any model assumptions. In QCD sum rule approach, a correlation function of an interpolating field is calculated by the use of the operator product expansion (OPE), and is compared with the spectral representation via dispersion relation. The sum rules relate hadron properties to the vacuum expectation values of QCD composite operators (condensates) such as  $\langle 0|\bar{q}q|0\rangle$ ,  $\langle 0|(\alpha_s/\pi)G^2|0\rangle$  and so on. From the relation, one can understand hadron properties in terms of the structure of the QCD vacuum.

The paper is organized as follows. In the second section, we formulate the general method for deriving the QCD sum rules for positive and negative parity baryons with  $J = 3/2$ . Then, in the third section, we apply the method to constructing the sum rules for the pentaquark with  $IJ^P = 0\frac{3}{2}^\pm$  and that with  $1\frac{3}{2}^\pm$ . From the obtained sum rules, we show the numerical results, discuss whether those states of pentaquark can exist, and evaluate their masses in the fourth section. In the fifth section, we discuss whether they can be narrow states and the relation with the investigation by other approaches, and finally summarize the paper.

## II. QCD SUM RULE FOR POSITIVE AND NEGATIVE PARITY BARYONS WITH $J = 3/2$

In this section, we formulate the QCD sum rule for positive and negative parity states of  $J = 3/2$  baryons.

The correlation function from which we derive the QCD sum rule is

$$\Pi_{\mu\nu}(p) = -i \int d^4x \exp(ipx) \langle 0 | T [\eta_\mu(x) \bar{\eta}_\nu(0)] | 0 \rangle, \quad (1)$$

where  $\eta_\mu$  is an interpolating field that couples to baryon states with  $J = 3/2$ .  $\eta_\mu$  is constructed with quark (and gluon) fields so as to have the quantum numbers of the baryon

which we want to know about. The correlation function, Eq.(1), has various tensor structures,

$$\begin{aligned}\Pi_{\mu\nu}(p) = & \left[ \Pi_1(p^2)g_{\mu\nu}p' + \Pi_2(p^2)\gamma_\mu\gamma_\nu p' + \Pi_3(p^2)\gamma_\mu p_\nu + \Pi_4(p^2)\gamma_\nu p_\mu + \Pi_5(p^2)p_\mu p_\nu p' \right] \\ & + \left[ \Pi_6(p^2)g_{\mu\nu} + \Pi_7(p^2)\gamma_\mu\gamma_\nu + \Pi_8(p^2)\gamma_\mu p_\nu p' + \Pi_9(p^2)\gamma_\nu p_\mu p' + \Pi_{10}(p^2)p_\mu p_\nu \right].\end{aligned}\quad (2)$$

We consider the terms proportional to  $g_{\mu\nu}$ :

$$\Pi(p) \equiv p' \Pi_1(p^2) + \Pi_6(p^2), \quad (3)$$

since only the  $J = 3/2$  states contribute to these terms. Other terms include the contribution from not only  $J = 3/2$  states but also  $J = 1/2$  states [26].

We can relate the correlation function with the spectral function via Lehman representation,

$$\Pi(p_0, \mathbf{p}) = \int_{-\infty}^{\infty} \frac{\rho(p'_0, \mathbf{p})}{p_0 - p'_0} dp'_0, \quad (4)$$

where  $\rho(p_0, \mathbf{p}) \equiv (1/\pi)\text{Im}\Pi(p_0 + i\epsilon, \mathbf{p})$  is the spectral function. On the other hand, in the deep Euclid region,  $p_0^2 \rightarrow -\infty$ , the correlation function can be evaluated by an OPE. Then the correlation function is expressed as a sum of various vacuum condensates. Using the analyticity, we obtain a relation between the imaginary part of the correlation function evaluated by an OPE,  $\rho^{\text{OPE}}(p_0, \mathbf{p})$ , and the spectral function as

$$\int_{-\infty}^{\infty} dp_0 \rho^{\text{OPE}}(p_0, \mathbf{p}) W(p_0) = \int_{-\infty}^{\infty} dp_0 \rho(p_0, \mathbf{p}) W(p_0), \quad (5)$$

where  $W(p_0)$  is an analytic function of  $p_0$ . Eq.(5) is a general form of the QCD sum rule. By properly parameterizing  $\rho(p_0, \mathbf{p})$ , we obtain QCD sum rules for physical quantities in  $\rho(p_0, \mathbf{p})$ .

Let us first look at the spectral function,  $\rho(p_0, \mathbf{p})$ . We consider the interpolating field with negative parity. The interpolating field couples to positive parity states as well as negative parity states [27]. Accordingly, both of the parity states contribute to  $\Pi(p)$  as physical intermediate states. The expression of  $\Pi(p)$  in terms of the physical states reads [26],

$$\Pi(p) = \sum_n \left[ -|\lambda_-^{(n)}|^2 \frac{p' + m_-^{(n)}}{p^2 - m_-^{(n)2}} - |\lambda_+^{(n)}|^2 \frac{p' - m_+^{(n)}}{p^2 - m_+^{(n)2}} \right], \quad (6)$$

where  $m_{+,-}^{(n)}$  are the masses of the  $n$ -th positive and negative parity states, respectively, and  $\lambda_{+,-}^{(n)}$  the coupling strengths of the interpolating field with the positive and negative parity

states. In Eq.(6), the widths of the physical states were neglected. The spectral function in the rest frame,  $\mathbf{p} = \mathbf{0}$ , can be decomposed into two parts,

$$\rho(p_0) = P_+ \rho_-(p_0) + P_- \rho_+(p_0), \quad (7)$$

where  $P_{\pm} = (\gamma_0 \pm 1)/2$  and  $\rho_{\mp}(p_0)$  are expressed as

$$\rho_-(p_0) = \sum_n \left[ |\lambda_-^{(n)}|^2 \delta(p_0 - m_-^{(n)}) + |\lambda_+^{(n)}|^2 \delta(p_0 + m_+^{(n)}) \right], \quad (8)$$

$$\rho_+(p_0) = \sum_n \left[ |\lambda_+^{(n)}|^2 \delta(p_0 - m_+^{(n)}) + |\lambda_-^{(n)}|^2 \delta(p_0 + m_-^{(n)}) \right]. \quad (9)$$

Here, for later use, we note that the spectral function for the interpolating field with positive parity is given by interchanging  $\rho_-(p_0)$  and  $\rho_+(p_0)$  in Eq.(7),

$$\rho(p_0) = P_+ \rho_+(p_0) + P_- \rho_-(p_0), \quad (\text{for } \eta_\mu \text{ with positive parity}). \quad (10)$$

Next, we construct the sum rule for negative parity states and that for positive parity. We apply the projection operator  $P_{\pm}$  to Eq.(5) for  $\mathbf{p} = \mathbf{0}$ . Then we obtain

$$\int_{-\infty}^{\infty} dp_0 \rho_{\mp}^{\text{OPE}}(p_0) W(p_0) = \int_{-\infty}^{\infty} dp_0 \rho_{\mp}(p_0) W(p_0). \quad (11)$$

Note that in Eq.(11) the contributions from the positive and negative parity states are mixed since, as can be seen from Eqs.(8) and (9), each of  $\rho_-(p_0)$  and  $\rho_+(p_0)$  contains the contributions from both of the parity states. What we want to do is to separate the negative and the positive parity contributions from Eqs.(11). The following procedure of the parity projection is essentially equivalent to that in Ref.[28].

If the expressions of the spectral functions calculated by an OPE,  $\rho_{\mp}^{\text{OPE}}(p_0)$ , are separable into  $\rho_{\mp}^{\text{OPE}}(p_0 > 0)$  and  $\rho_{\mp}^{\text{OPE}}(p_0 < 0)$ , we can independently construct the sum rule from the  $p_0 > 0$  part of the correlation function and that from the  $p_0 < 0$  part [28]. The sum rules obtained from the  $p_0 > 0$  part are

$$\int_0^{\infty} dp_0 \rho_-^{\text{OPE}}(p_0) W(p_0) = \int_0^{\infty} dp_0 \rho_-(p_0) W(p_0), \quad (12)$$

$$\int_0^{\infty} dp_0 \rho_+^{\text{OPE}}(p_0) W(p_0) = \int_0^{\infty} dp_0 \rho_+(p_0) W(p_0). \quad (13)$$

Eq.(12) is the sum rule for the negative parity state since only the negative parity states contribute to  $\rho_-(p_0)$  for  $p_0 > 0$ . On the other hand, Eq.(13) is the sum rule for the positive parity state since  $\rho_+(p_0)$  for  $p_0 > 0$  contains only the positive parity states (see Eqs.(8) and (9)).

A comment is in order here. In order to separate the positive and negative parity states in the sum rule as Eqs.(12) and (13), it is necessary that  $\rho_{\mp}^{\text{OPE}}(p_0)$  are separable into  $\rho_{\mp}^{\text{OPE}}(p_0 > 0)$  and  $\rho_{\mp}^{\text{OPE}}(p_0 < 0)$  as mentioned above. In general,  $\rho_{\mp}^{\text{OPE}}(p_0)$  are not separable [29] because the OPE terms depend on  $(p_0)^n [\theta(p_0) - \theta(-p_0)]$  or  $\delta^{(n)}(p_0)$ . However, as will be seen in the next section,  $\rho_{\mp}^{\text{OPE}}(p_0)$  for pentaquark is separable as long as we truncate the OPE at certain order since  $\rho_{\mp}^{\text{OPE}}(p_0)$  up to dimension 7 operator contain only the  $(p_0)^n [\theta(p_0) - \theta(-p_0)]$  terms. We can thus derive the sum rule for each parity state of the pentaquark as Eqs.(12) and (13).

In Eqs.(12) and (13), we parameterize  $\rho_{\mp}(p_0)$  for  $p_0 > 0$  with a pole plus continuum contribution,

$$\rho_{\mp}(p_0) = |\lambda_{\mp}|^2 \delta(p_0 - m_{\mp}) + \theta(p_0 - \omega_{\mp}) \rho_{\mp}^{\text{OPE}}(p_0), \quad \text{for } p_0 > 0, \quad (14)$$

where  $|\lambda_{\mp}|^2$  and  $m_{\mp}$  are the pole residues and the masses of the lowest states, respectively.  $\omega_{\mp}$  denote the effective continuum threshold. Substituting Eq.(14) into the right-hand sides of Eqs.(12) and (13), we obtain the following sum rules,

$$\int_0^{\omega_{\mp}} dp_0 \rho_{\mp}^{\text{OPE}}(p_0) (p_0)^n \exp\left(-\frac{p_0^2}{M^2}\right) = (m_{\mp})^n |\lambda_{\mp}|^2 \exp\left(-\frac{m_{\mp}^2}{M^2}\right). \quad (15)$$

Here we have chosen the weight function as  $W(p_0) = (p_0)^n \exp(-p_0^2/M^2)$ , where  $n$  is an arbitrary positive integer. The parameter  $M$  is called “Borel mass”. By introducing such weight function, one can improve the convergence of the OPE and simultaneously suppress the continuum contribution.

From Eq.(15) for  $n = 0$ , we obtain the sum rule for the pole residues  $|\lambda_{\mp}|^2$ ,

$$|\lambda_{\mp}|^2 \exp\left(-\frac{m_{\mp}^2}{M^2}\right) = \int_0^{\omega_{\mp}} dp_0 \rho_{\mp}^{\text{OPE}}(p_0) \exp\left(-\frac{p_0^2}{M^2}\right). \quad (16)$$

The masses can be extracted from the ratio of Eq.(15) for  $n = 0$  and  $n = 2$ ,

$$m_{\mp}^2 = \frac{\int_0^{\omega_{\mp}} dp_0 \rho_{\mp}^{\text{OPE}}(p_0) (p_0)^2 \exp\left(-\frac{p_0^2}{M^2}\right)}{\int_0^{\omega_{\mp}} dp_0 \rho_{\mp}^{\text{OPE}}(p_0) \exp\left(-\frac{p_0^2}{M^2}\right)}. \quad (17)$$

### III. QCD SUM RULES FOR THE $IJ^P = 0\frac{3}{2}^{\pm}$ AND $1\frac{3}{2}^{\pm}$ STATES OF PENTAQUARK

Utilizing the method formulated in the previous section, we derive the QCD sum rules for the  $IJ^P = 0\frac{3}{2}^{\pm}$  and  $1\frac{3}{2}^{\pm}$  states of the pentaquark baryons.

Our first task is to construct the interpolating fields for the spin-3/2 pentaquark baryons. There are various ways of constructing interpolating fields. In this paper, we examine two independent interpolating fields for each isospin state. Then, the correlation functions of the interpolating fields are evaluated by the use of OPE. For each isospin state, we choose the interpolating field which has better convergence of OPE among the two, and construct the sum rules.

### A. Interpolating field

The interpolating fields for  $I = 0$  state which we employ are those proposed by Sasaki [13],

$$\eta_{1,\mu}^{I=0}(x) = \epsilon_{cfg} \left[ \epsilon_{abc} u_a^T(x) C \gamma_5 d_b(x) \right] \left[ \epsilon_{def} u_d^T(x) C \gamma_\mu \gamma_5 d_{e(x)} \right] C \bar{s}_g^T(x), \quad (18)$$

$$\eta_{2,\mu}^{I=0}(x) = \epsilon_{cfg} \left[ \epsilon_{abc} u_a^T(x) C d_b(x) \right] \left[ \epsilon_{def} u_d^T(x) C \gamma_\mu \gamma_5 d_{e(x)} \right] \gamma_5 C \bar{s}_g^T(x), \quad (19)$$

where  $u$ ,  $d$  and  $s$  are up, down and strange quark fields, respectively, roman indices  $a, b, \dots$  are color,  $C = i\gamma^2\gamma^0$  is the charge conjugation matrix, and  $T$  transpose. Eq.(18) consists of two diquark fields,  $S_c(x) \equiv \epsilon_{abc} u_a^T(x) C \gamma_5 d_b(x)$  and  $V_f(x) \equiv \epsilon_{def} u_d^T(x) C \gamma_\mu \gamma_5 d_{e(x)}$ , and an anti-strange quark field,  $C \bar{s}_g^T(x)$ . The color structure is  $\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}}$ .  $S_c(x)$  is a color  $\bar{\mathbf{3}}$  scalar diquark operator with  $I = 0$ , which corresponds to the  ${}^1S_0$  state of the  $I = 0$   $ud$ -diquark system.  $V_f(x)$  is a color  $\bar{\mathbf{3}}$  vector diquark with  $I = 0$ , and is assigned to  ${}^3P_1$  of the  $I = 0$   $ud$ -diquark system. Eq.(18) is therefore totally  $I = 0$ , and hence, one can confirm that Eq.(18) can create the states with  $IJ = 0\frac{3}{2}$ . The parity of Eq.(18) is positive since the intrinsic parity of  $C \bar{s}_g^T(x)$  is negative.

Alternatively, we can construct the interpolating field for  $I = 0$  by using a pseudo-scalar diquark operator,  $P_c(x) \equiv \epsilon_{abc} u_a^T(x) C d_b(x)$ , instead of the scalar diquark.  $P_c(x)$  corresponds to the  ${}^3P_0$  state of the  $I = 0$   $ud$ -diquark system. In Eq.(19), we multiplied  $C \bar{s}_g^T(x)$  by  $\gamma_5$  to make the total parity positive. Clearly, Eq.(18) can also couple with  $IJ = 0\frac{3}{2}$  states.

In a quite similar way, we can construct the following two interpolating fields for the  $I = 1$  states, using the scalar or pseudo-scalar diquark and an axial-vector diquark operator [24],

$$\eta_{1,\mu}^{I=1}(x) = \epsilon_{cfg} \left[ \epsilon_{abc} u_a^T(x) C \gamma_5 d_b(x) \right] \left[ \epsilon_{def} u_d^T(x) C \gamma_\mu u_e(x) \right] C \bar{s}_g^T(x), \quad (20)$$

$$\eta_{2,\mu}^{I=1}(x) = \epsilon_{cfg} \left[ \epsilon_{abc} u_a^T(x) C d_b(x) \right] \left[ \epsilon_{def} u_d^T(x) C \gamma_\mu u_e(x) \right] \gamma_5 C \bar{s}_g^T(x), \quad (21)$$

where  $A_f(x) \equiv \epsilon_{def} u_d^T(x) C \gamma_\mu u_e(x)$  is a color  $\bar{\mathbf{3}}$  axial-vector diquark with  $I = 1$ .  $A_f(x)$  corresponds to  ${}^3S_1$  of the  $I = 1$   $ud$ -diquark system. One can easily see that either of Eq.(20) and Eq.(21) contains  $IJ = 1\frac{3}{2}$  states. It should be noted that total parity of Eqs.(20) and (21) is negative. We remark that the way of constructing Eq.(20) is based on the structure of the  $1\frac{3}{2}^-$  pentaquark state suggested from a quark model [19].

## B. OPE

We now evaluate the correlation functions of the interpolating fields, Eqs.(18)~(21),

$$\begin{aligned} \Pi_{j,\mu\nu}^I(p) &= -i \int d^4x \exp(ipx) \langle 0 | T \left[ \eta_{j,\mu}^I(x) \bar{\eta}_{j,\nu}^I(0) \right] | 0 \rangle, \\ &= g_{\mu\nu} \Pi_j^I(p) + (\text{other tensor structures}), \quad (I = 0, 1, j = 1, 2), \end{aligned} \quad (22)$$

by the use of OPE. We take into account the terms up to dimension 7 operators and neglect the masses of up and down quarks. The spectral function of  $\Pi_j^I(p)$  for  $\mathbf{p}=0$ ,  $\rho_j^I(p_0)$ , is parametrised in terms of the chirality conserving term and the violating term, which we denote by  $A_j^I(p_0)$  and  $B_j^I(p_0)$ , respectively, as

$$\rho_j^I(p_0) = \gamma_0 A_j^I(p_0) + B_j^I(p_0). \quad (23)$$

We show the results of the OPE of  $A_j^I(p_0)$  and  $B_j^I(p_0)$  in the following.

When we use the interpolating field  $\eta_{1,\mu}^{I=0}(x)$ , we obtain

$$\begin{aligned} A_1^{I=0}(p_0) &= \left[ a_0 \cdot (p_0)^{11} + a_1 \cdot \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle (p_0)^7 \right. \\ &\quad + a_2 \cdot m_s \langle 0 | \bar{s}s | 0 \rangle (p_0)^7 + a_3 \cdot m_s g \langle 0 | \bar{s}\sigma^{\mu\nu} (\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle (p_0)^5 \\ &\quad \left. + a_4 \cdot \langle 0 | \bar{q}q | 0 \rangle^2 (p_0)^5 \right] \times [\theta(p_0) - \theta(-p_0)], \end{aligned} \quad (24)$$

$$\begin{aligned} B_1^{I=0}(p_0) &= \left[ b_0 \cdot m_s (p_0)^{10} + b_1 \cdot \langle 0 | \bar{s}s | 0 \rangle (p_0)^8 \right. \\ &\quad + b_2 \cdot g \langle 0 | \bar{s}\sigma^{\mu\nu} (\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle (p_0)^6 \\ &\quad + b_3 \cdot m_s \langle 0 | \bar{q}q | 0 \rangle^2 (p_0)^4 + b_4 \cdot \langle 0 | \bar{s}s | 0 \rangle \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle (p_0)^4 \\ &\quad \left. \times [\theta(p_0) - \theta(-p_0)] \right], \end{aligned} \quad (25)$$

where the coefficients  $a_i$  and  $b_i$  are given by

$$\begin{aligned} a_0 &= \frac{1}{5^2 \cdot 3^2 \cdot 2^{19} \pi^8}, \quad a_1 = \frac{-7}{5 \cdot 3^4 \cdot 2^{18} \pi^6}, \quad a_2 = \frac{1}{3^3 \cdot 2^{13} \pi^6}, \\ a_3 &= \frac{-37}{5 \cdot 3^2 \cdot 2^{16} \pi^6}, \quad a_4 = \frac{1}{5 \cdot 3^2 \cdot 2^9 \pi^4}, \\ b_0 &= \frac{1}{7 \cdot 5^2 \cdot 3 \cdot 2^{15} \pi^8}, \quad b_1 = \frac{-1}{3^3 \cdot 2^{14} \pi^6}, \quad b_2 = \frac{101}{5 \cdot 3^3 \cdot 2^{16} \pi^6}, \end{aligned} \quad (26)$$

$$b_3 = \frac{1}{3^2 \cdot 2^9 \pi^4}, \quad b_4 = \frac{-7}{3^3 \cdot 2^{15} \pi^4}. \quad (27)$$

In Eqs.(24) and (25),  $m_s$  is the strange quark mass,  $g$  is the gauge coupling constant and  $\alpha_s = g^2/(4\pi)$ .  $q \equiv u = d$ ,  $G_{\mu\nu}^a$  is the strength of gluon field and  $\langle 0|\mathcal{O}|0\rangle$  denotes the vacuum expectation value of the operator  $\mathcal{O}$ .

The results when we employ  $\eta_{2,\mu}^{I=0}(x)$  are as follows,

$$\begin{aligned} A_2^{I=0}(p_0) = & \left[ a_0 \cdot (p_0)^{11} + a_1 \cdot \langle 0|(\alpha_s/\pi)G^{a\mu\nu}G_{\mu\nu}^a|0\rangle(p_0)^7 \right. \\ & + a_2 \cdot m_s \langle 0|\bar{s}s|0\rangle(p_0)^7 + a_3 \cdot m_s g \langle 0|\bar{s}\sigma^{\mu\nu}(\lambda^a/2)G_{\mu\nu}^a s|0\rangle(p_0)^5 \\ & \left. - 9 \cdot a_4 \cdot \langle 0|\bar{q}q|0\rangle^2(p_0)^5 \right] \times [\theta(p_0) - \theta(-p_0)], \end{aligned} \quad (28)$$

$$\begin{aligned} B_2^{I=0}(p_0) = & \left[ -b_0 \cdot m_s(p_0)^{10} - b_1 \cdot \langle 0|\bar{s}s|0\rangle(p_0)^8 \right. \\ & - b_2 \cdot g \langle 0|\bar{s}\sigma^{\mu\nu}(\lambda^a/2)G_{\mu\nu}^a s|0\rangle(p_0)^6 \\ & \left. + 7 \cdot b_3 \cdot m_s \langle 0|\bar{q}q|0\rangle^2(p_0)^4 - b_4 \cdot \langle 0|\bar{s}s|0\rangle \langle 0|(\alpha_s/\pi)G^{a\mu\nu}G_{\mu\nu}^a|0\rangle(p_0)^4 \right] \\ & \times [\theta(p_0) - \theta(-p_0)], \end{aligned} \quad (29)$$

Let us compare the results for  $\eta_{1,\mu}^{I=0}$ , Eqs.(24) and (25), with those for  $\eta_{2,\mu}^{I=0}$ , Eqs.(28) and (29). We can see that the OPE convergence of the correlation function of  $\eta_{2,\mu}^{I=0}$  is clearly slower than that of  $\eta_{1,\mu}^{I=0}$  since the contributions of dimension 6 ( $\langle 0|\bar{q}q|0\rangle^2$ ) and dimension 7 ( $m_s \langle 0|\bar{q}q|0\rangle^2$ ) terms in Eqs.(28) and (29) are larger than those in Eqs.(24) and (25). Therefore, we adopt  $\eta_{1,\mu}^{I=0}$  for deriving the sum rules.

Next, we show the results of OPE for  $I = 1$  channel. When using  $\eta_{1,\mu}^{I=1}(x)$ , we obtain

$$\begin{aligned} A_1^{I=1}(p_0) = & \left[ a_0 \cdot (p_0)^{11} + a_1 \cdot \langle 0|(\alpha_s/\pi)G^{a\mu\nu}G_{\mu\nu}^a|0\rangle(p_0)^7 \right. \\ & + a_2 \cdot m_s \langle 0|\bar{s}s|0\rangle(p_0)^7 + a_3 \cdot m_s g \langle 0|\bar{s}\sigma^{\mu\nu}(\lambda^a/2)G_{\mu\nu}^a s|0\rangle(p_0)^5 \\ & \left. + 9 \cdot a_4 \cdot \langle 0|\bar{q}q|0\rangle^2(p_0)^5 \right] \times [\theta(p_0) - \theta(-p_0)], \end{aligned} \quad (30)$$

$$\begin{aligned} B_1^{I=1}(p_0) = & \left[ b_0 \cdot m_s(p_0)^{10} + b_1 \cdot \langle 0|\bar{s}s|0\rangle(p_0)^8 \right. \\ & + b_2 \cdot g \langle 0|\bar{s}\sigma^{\mu\nu}(\lambda^a/2)G_{\mu\nu}^a s|0\rangle(p_0)^6 \\ & \left. + 7 \cdot b_3 \cdot m_s \langle 0|\bar{q}q|0\rangle^2(p_0)^4 + b_4 \cdot \langle 0|\bar{s}s|0\rangle \langle 0|(\alpha_s/\pi)G^{a\mu\nu}G_{\mu\nu}^a|0\rangle(p_0)^4 \right] \\ & \times [\theta(p_0) - \theta(-p_0)], \end{aligned} \quad (31)$$

while the results for  $\eta_{2,\mu}^{I=1}(x)$  are as follows,

$$\begin{aligned} A_2^{I=1}(p_0) = & \left[ a_0 \cdot (p_0)^{11} + a_1 \cdot \langle 0|(\alpha_s/\pi)G^{a\mu\nu}G_{\mu\nu}^a|0\rangle(p_0)^7 \right. \\ & + a_2 \cdot m_s \langle 0|\bar{s}s|0\rangle(p_0)^7 + a_3 \cdot m_s g \langle 0|\bar{s}\sigma^{\mu\nu}(\lambda^a/2)G_{\mu\nu}^a s|0\rangle(p_0)^5 \end{aligned}$$

$$+a_4 \cdot \langle 0|\bar{q}q|0\rangle^2(p_0)^5] \times [\theta(p_0) - \theta(-p_0)], \quad (32)$$

$$\begin{aligned} B_2^{I=1}(p_0) = & \left[ -b_0 \cdot m_s(p_0)^{10} - b_1 \cdot \langle 0|\bar{s}s|0\rangle(p_0)^8 \right. \\ & - b_2 \cdot g \langle 0|\bar{s}\sigma^{\mu\nu}(\lambda^a/2)G_{\mu\nu}^a s|0\rangle(p_0)^6 \\ & \left. - b_3 \cdot m_s \langle 0|\bar{q}q|0\rangle^2(p_0)^4 - b_4 \cdot \langle 0|\bar{s}s|0\rangle \langle 0|(\alpha_s/\pi)G^{a\mu\nu}G_{\mu\nu}^a|0\rangle(p_0)^4 \right] \\ & \times [\theta(p_0) - \theta(-p_0)]. \end{aligned} \quad (33)$$

Comparing Eqs.(30) and (31) with Eqs.(32) and (33), we see that  $\eta_{2,\mu}^{I=1}$  has better convergence than  $\eta_{1,\mu}^{I=1}$  since the contributions of dimension 6 ( $\langle 0|\bar{q}q|0\rangle^2$ ) and dimension 7 ( $m_s \langle 0|\bar{q}q|0\rangle^2$ ) terms in Eqs.(30) and (31) are larger than those in Eqs.(32) and (33). From this reason, we employ  $\eta_{2,\mu}^{I=1}$  to derive the sum rules for  $I = 1$  states.

Note here that Eq.(32) exactly coincides with Eq.(24). Also, Eq.(33) and Eq.(25) coincide except for the overall sign.

### C. Sum rules for $I = 0$ states

Using the OPE results, Eqs.(24)~(25), we derive the QCD sum rules for the  $IJ^P = 0\frac{3}{2}^\pm$  states of the pentaquark. From Eq.(10),  $\rho_\pm^{\text{OPE}}(p_0)$  for  $\eta_\mu^{I=0}$  is written in terms of  $A(p_0)$  and  $B(p_0)$  as,

$$\rho_\pm^{\text{OPE}}(p_0) = A^{I=0}(p_0) \pm B^{I=0}(p_0), \quad (34)$$

We substitute Eq.(34) with Eqs.(24) and (25) into the right hand side of Eq.(16) to obtain the sum rules,

$$\begin{aligned} & |\lambda_\pm^{I=0}|^2 \exp \left[ -\frac{(m_\pm^{I=0})^2}{M^2} \right] \\ = & \left[ a_0 \cdot f(M, \omega_\pm^{I=0}; 11) + a_1 \cdot \langle 0|(\alpha_s/\pi)G^{a\mu\nu}G_{\mu\nu}^a|0\rangle f(M, \omega_\pm^{I=0}; 7) \right. \\ & + a_2 \cdot m_s \langle 0|\bar{s}s|0\rangle f(M, \omega_\pm^{I=0}; 7) + a_3 \cdot m_s g \langle 0|\bar{s}\sigma^{\mu\nu}(\lambda^a/2)G_{\mu\nu}^a s|0\rangle f(M, \omega_\pm^{I=0}; 5) \\ & + a_4 \cdot \langle 0|\bar{q}q|0\rangle^2 f(M, \omega_\pm^{I=0}; 5) \left. \right] \\ & \pm \left[ b_0 \cdot m_s f(M, \omega_\pm^{I=0}; 10) + b_1 \cdot \langle 0|\bar{s}s|0\rangle f(M, \omega_\pm^{I=0}; 8) \right. \\ & + b_2 \cdot g \langle 0|\bar{s}\sigma^{\mu\nu}(\lambda^a/2)G_{\mu\nu}^a s|0\rangle f(M, \omega_\pm^{I=0}; 6) + b_3 \cdot m_s \langle 0|\bar{q}q|0\rangle^2 f(M, \omega_\pm^{I=0}; 4) \\ & \left. + b_4 \cdot \langle 0|\bar{s}s|0\rangle \langle 0|(\alpha_s/\pi)G^{a\mu\nu}G_{\mu\nu}^a|0\rangle f(M, \omega_\pm^{I=0}; 4) \right], \end{aligned} \quad (35)$$

where  $|\lambda_{\pm}^I|^2$ ,  $m_{\pm}^I$  and  $\omega_{\pm}^I$  are the pole residues, the masses and the effective continuum threshold with the isospin  $I$  channel, respectively.  $f(M, \omega; n)$  is the integral defined by

$$f(M, \omega; n) \equiv \int_0^{\omega} dp_0(p_0)^n \exp(-\frac{p_0^2}{M^2}). \quad (36)$$

The sum rules for the masses are obtained by substituting the OPE into Eq.(17) as

$$\begin{aligned} (m_{\pm}^{I=0})^2 = & \left\{ \left[ a_0 \cdot f(M, \omega_{\pm}^{I=0}; 13) + a_1 \cdot \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=0}; 9) \right. \right. \\ & + a_2 \cdot m_s \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 9) + a_3 \cdot m_s g \langle 0 | \bar{s}\sigma^{\mu\nu} (\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 7) \\ & + a_4 \cdot \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=0}; 7) \left. \right] \\ & \pm \left[ b_0 \cdot m_s f(M, \omega_{\pm}^{I=0}; 12) + b_1 \cdot \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 10) \right. \\ & + b_2 \cdot g \langle 0 | \bar{s}\sigma^{\mu\nu} (\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 8) + b_3 \cdot m_s \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=0}; 6) \\ & \left. \left. + b_4 \cdot \langle 0 | \bar{s}s | 0 \rangle \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=0}; 6) \right] \right\} \\ & / \left\{ \left[ a_0 \cdot f(M, \omega_{\pm}^{I=0}; 11) + a_1 \cdot \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=0}; 7) \right. \right. \\ & + a_2 \cdot m_s \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 7) + a_3 \cdot m_s g \langle 0 | \bar{s}\sigma^{\mu\nu} (\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 5) \\ & + a_4 \cdot \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=0}; 5) \left. \right] \\ & \pm \left[ b_0 \cdot m_s f(M, \omega_{\pm}^{I=0}; 10) + b_1 \cdot \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 8) \right. \\ & + b_2 \cdot g \langle 0 | \bar{s}\sigma^{\mu\nu} (\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 6) + b_3 \cdot m_s \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=0}; 4) \\ & \left. \left. + b_4 \cdot \langle 0 | \bar{s}s | 0 \rangle \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=0}; 4) \right] \right\}. \end{aligned} \quad (37)$$

It should be noted that the mass splitting between positive and negative parity states is due to the  $\pm$  sign of the terms containing  $m_s$  and the condensates of chiral odd operators [28].

#### D. Sum rules for $I = 1$ states

From Eq.(7), we see that  $\rho_{\pm}^{\text{OPE}}(p_0)$  for  $\eta_{\mu}^{I=1}$  is written in terms of  $A(p_0)$  and  $B(p_0)$  as,

$$\rho_{\pm}^{\text{OPE}}(p_0) = A^{I=1}(p_0) \mp B^{I=1}(p_0). \quad (38)$$

The sum rules for the  $IJ^P = 1\frac{3}{2}^{\pm}$  states are obtained by using Eq.(38) with Eqs.(32) and (33),

$$\begin{aligned} & |\lambda_{\pm}^{I=1}|^2 \exp \left[ -\frac{(m_{\pm}^{I=1})^2}{M^2} \right] \\ & = \left[ a_0 \cdot f(M, \omega_{\pm}^{I=1}; 11) + a_1 \cdot \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=1}; 7) \right. \\ & \left. + a_2 \cdot m_s \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=1}; 7) + a_3 \cdot m_s g \langle 0 | \bar{s}\sigma^{\mu\nu} (\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=1}; 5) \right] \end{aligned}$$

$$\begin{aligned}
& + a_4 \cdot \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=1}; 5) \Big] \\
& \mp \left[ -b_0 \cdot m_s f(M, \omega_{\pm}^{I=1}; 10) - b_1 \cdot \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=1}; 8) \right. \\
& - b_2 \cdot g \langle 0 | \bar{s}\sigma^{\mu\nu}(\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=1}; 6) - b_3 \cdot m_s \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=1}; 4) \\
& \left. - b_4 \cdot \langle 0 | \bar{s}s | 0 \rangle \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=1}; 4) \right]. \tag{39}
\end{aligned}$$

The mass sum rule for  $I = 1$  are as follows,

$$\begin{aligned}
(m_{\pm}^{I=1})^2 = & \left\{ \left[ a_0 \cdot f(M, \omega_{\pm}^{I=0}; 13) + a_1 \cdot \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=0}; 9) \right. \right. \\
& + a_2 \cdot m_s \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 9) + a_3 \cdot m_s g \langle 0 | \bar{s}\sigma^{\mu\nu}(\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 7) \\
& + a_4 \cdot \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=0}; 7) \Big] \\
& \mp \left[ -b_0 \cdot m_s f(M, \omega_{\pm}^{I=0}; 12) - b_1 \cdot \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 10) \right. \\
& - b_2 \cdot g \langle 0 | \bar{s}\sigma^{\mu\nu}(\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 8) - b_3 \cdot m_s \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=0}; 6) \\
& \left. \left. - b_4 \cdot \langle 0 | \bar{s}s | 0 \rangle \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=0}; 6) \right] \right\} \\
& / \left\{ \left[ a_0 \cdot f(M, \omega_{\pm}^{I=0}; 11) + a_1 \cdot \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=0}; 7) \right. \right. \\
& + a_2 \cdot m_s \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 7) + a_3 \cdot m_s g \langle 0 | \bar{s}\sigma^{\mu\nu}(\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 5) \\
& + a_4 \cdot \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=0}; 5) \Big] \\
& \mp \left[ -b_0 \cdot m_s f(M, \omega_{\pm}^{I=0}; 10) - b_1 \cdot \langle 0 | \bar{s}s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 8) \right. \\
& - b_2 \cdot g \langle 0 | \bar{s}\sigma^{\mu\nu}(\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle f(M, \omega_{\pm}^{I=0}; 6) - b_3 \cdot m_s \langle 0 | \bar{q}q | 0 \rangle^2 f(M, \omega_{\pm}^{I=0}; 4) \\
& \left. \left. - b_4 \cdot \langle 0 | \bar{s}s | 0 \rangle \langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle f(M, \omega_{\pm}^{I=0}; 4) \right] \right\}. \tag{40}
\end{aligned}$$

#### IV. NUMERICAL RESULTS

In this section, we show the numerical results obtained from the sum rules for  $IJ^P = 0\frac{3}{2}^\pm$  states, Eqs.(35) and (37), and those for  $1\frac{3}{2}^\pm$ , Eqs.(39) and (40). Throughout this paper, we use the QCD parameters of the standard values,  $\langle 0 | \bar{q}q | 0 \rangle = (-0.23 \text{ GeV})^3$ ,  $m_s = 0.12 \text{ GeV}$ ,  $\langle 0 | \bar{s}s | 0 \rangle = 0.8 \langle 0 | \bar{q}q | 0 \rangle$ ,  $g \langle 0 | \bar{s}\sigma^{\mu\nu}(\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle = (0.8 \text{ GeV}^2) \langle 0 | \bar{s}s | 0 \rangle$ ,  $\langle 0 | (\alpha_s/\pi) G^{a\mu\nu} G_{\mu\nu}^a | 0 \rangle = (0.33 \text{ GeV})^4$ .

##### A. $I = 0$ states

First, in order to see how good the convergence of OPE is, we show the contribution of each term in the right hand sides of the sum rules, Eq.(35), in Figs.1 and 2. We see that dimension 5 term, namely, mixed condensate  $g \langle 0 | \bar{s}\sigma^{\mu\nu}(\lambda^a/2) G_{\mu\nu}^a s | 0 \rangle$  gives dominant contribution. The terms higher than dimension 5 seems to decrease with increasing dimension;

the contributions of dimension 6 and 7 terms are about 50% and 30% of dimension 5 terms, respectively.

We plotted in Fig.3 the right-hand side of Eq.(35) as functions of the Borel mass,  $M$ . The pole residue  $|\lambda_{\mp}^{I=0}|^2$  must be positive otherwise the pole of the pentaquark is spurious. As can be seen in Fig.3, the right-hand side of Eq.(35) is positive. This suggests that the present QCD sum rule does not exclude the existence of the  $0\frac{3}{2}^{\pm}$  pentaquarks.

In Figs.4 and 5, we show the relative strength of the pole and the continuum contribution for negative and positive parity, respectively. The pole contribution is given by Eq.(35), while the continuum contribution is obtained by subtracting Eq.(35) from Eq.(35) with  $\omega_{\mp}^{I=0} = \infty$ . The percentage of the pole contribution must be as large as possible, but one usually accepts values around 50%. In the sum rule for negative parity, the pole contribution is sufficiently large at around  $M = 1.2$  GeV. On the other hand, in the sum rule for positive parity, the continuum contribution dominates the pole contribution, which implies that the sum rule for positive parity is less reliable than that for negative parity.

In Fig.6, we plotted the mass of the  $0\frac{3}{2}^-$  state,  $m_{-}^{I=0}$ , against the Borel mass which is obtained from Eq.(37). We see that there exists a region where the dependence on the Borel mass is weak and therefore the sum rule works. However, the curve depends on the choice of the effective continuum threshold,  $\omega_{-}^{I=0}$ . The continuum which comes from  $NK$  states starts up very gradually since it is  $D$ -wave in this channel. Around 1.8 GeV, the  $S$ -wave  $NK^*$  state opens and above the threshold it may give a large contribution to the continuum. We therefore choose the values of the effective continuum threshold as  $\omega_{-}^{I=0} = 1.8, 1.9, 2.0$  GeV. From the region of the curve weakly dependent of  $M$ ,  $m_{-}^{I=0}$  is predicted to be  $1.5 \sim 1.7$  GeV. A remarkable point here is that the mass is below the  $NK^*$  threshold (1.83 GeV). This means that the  $0\frac{3}{2}^-$  pentaquark can be extremely narrow, as discussed in the next section.

The mass of the  $0\frac{3}{2}^+$  state,  $m_{+}^{I=0}$ , against the Borel mass is shown in Fig.7. The continuum in this channel mainly comes from the  $P$ -wave  $NK$  states. It starts up more gradually than the  $S$ -wave  $NK$  continuum, but more rapidly than the  $D$ -wave continuum. In view of this point, we choose  $\omega_{+}^{I=0} = 1.7, 1.8, 1.9$  GeV. We see from Figs.6 and 7 that the dependence of the curve for  $0\frac{3}{2}^+$  on the effective continuum threshold is stronger than that for  $0\frac{3}{2}^-$ . This is natural since the relative strength of the pole for  $0\frac{3}{2}^+$  is weak compared with that for  $0\frac{3}{2}^-$ , as can be seen in Figs.4 and 5.

The masses of positive and negative parity states are nearly degenerate for  $0\frac{3}{2}$  pentaquark.

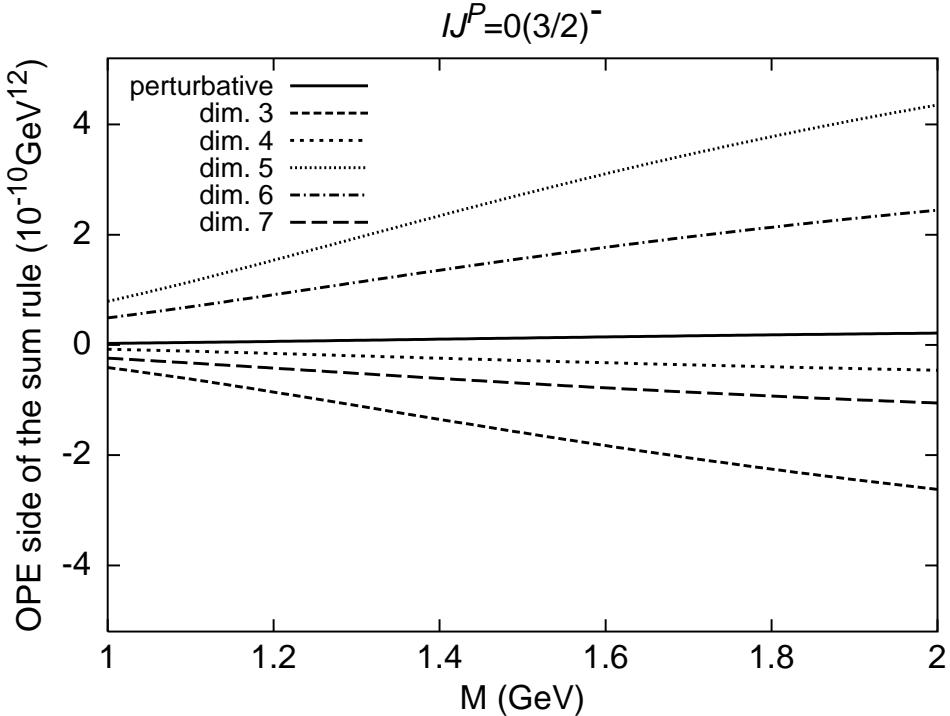


FIG. 1: Contributions of the terms in OPE side (the right hand side) of Eq.(35) for negative parity as functions of  $M$  with the effective continuum threshold  $\omega_-^{I=0} = 1.8$  GeV.

In the QCD sum rules for baryons, the parity splitting originates in the chiral odd term. For the  $0\frac{3}{2}^-$  pentaquark, the contribution of the chiral odd terms in the sum rule, Eq.(35), are small compared with the chiral even terms, which leads to the degeneracy of the positive and negative parity states.

### B. $I = 1$ states

We plot the contribution of each OPE term in the right hand sides of the sum rules, Eq.(39), in Figs.8 and 9. The behavior of the OPE for this channel is the same as that for  $I = 0$  channel.

The right-hand side of Eq.(39) as functions of  $M$  are plotted in Fig.10, which shows that the pole residues of the  $I = 1$  states are positive. This suggests that the poles of the  $1\frac{3}{2}^\pm$  pentaquark are not spurious.

In Figs.11 and 12, we show the relative strength of the pole and the continuum contribution for negative and positive parity, respectively. The pole contribution in the sum rule

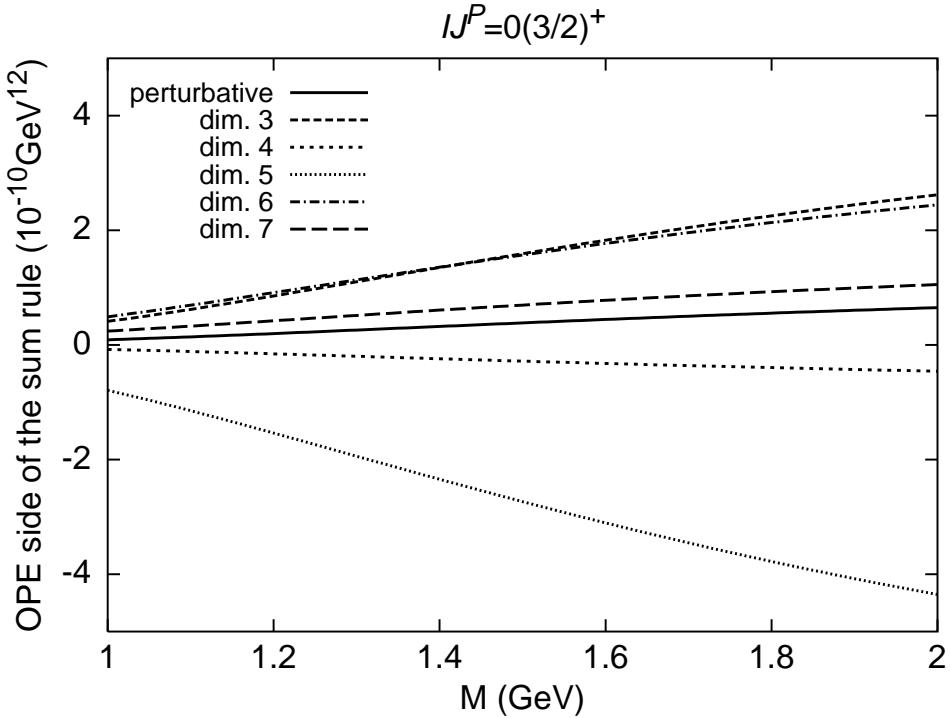


FIG. 2: Contributions of the terms in OPE side (the right hand side) of Eq.(35) for positive parity as functions of  $M$  with the effective continuum threshold  $\omega_+^{I=0} = 1.8$  GeV.

for negative parity is sufficiently large at around  $M = 1.2$  GeV, while for positive parity the continuum contribution dominates over the pole contribution, which implies that the reliability of the sum rule for positive parity is lower than that for negative parity.

In Fig.13, we plotted the mass of the  $1\frac{3}{2}^-$  state,  $m_-^{I=1}$ , against the Borel mass which is obtained from Eq.(40). Although the dependence on the Borel mass is weak, the curve depends on the choice of the effective continuum threshold  $\omega_-^{I=1}$ . The continuum which comes from  $D$ -wave  $NK$  states starts up very gradually. Around 1.7 GeV, the  $S$ -wave  $\Delta K$  state opens and above the threshold the continuum of the  $S$ -wave  $\Delta K$  may give a large contribution. Thus we choose  $\omega_-^{I=1} = 1.7, 1.8, 1.9$  GeV. From the stabilized region of the curve, we predict the mass to be  $1.4 \sim 1.6$  GeV, which is below the  $\Delta K$  threshold (1.73 GeV).

Let us turn to the sum rule for the positive parity state. The mass against the Borel mass is shown in Fig.14. The continuum in this channel mainly comes from the  $P$ -wave  $NK$  states. The effective continuum threshold is taken to be the same value as that for the  $0\frac{3}{2}^+$  state:  $\omega_+^{I=1} = 1.7, 1.8, 1.9$  GeV.

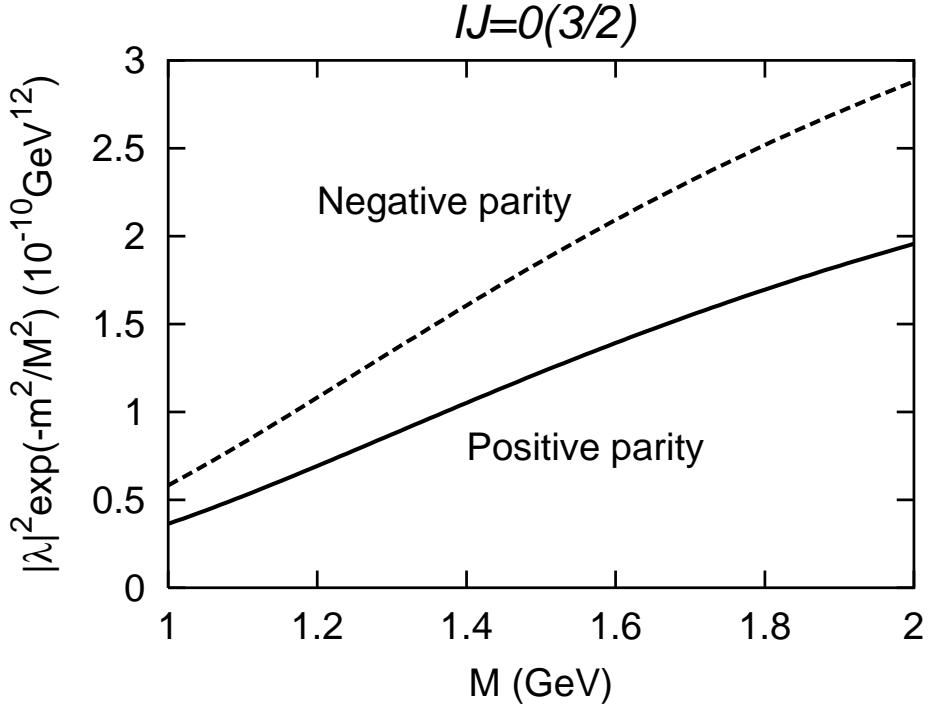


FIG. 3:  $|\lambda_{\mp}|^2 \exp(-\frac{m_{\mp}^2}{M^2})$  for  $I = 0$  as functions of Borel mass,  $M$ , obtained from Eq.(35) with the effective continuum threshold  $\omega_{\mp}^{I=0} = 1.8$  GeV.

As in the case for the  $IJ = 0\frac{3}{2}$  pentaquark, positive and negative parity states are nearly degenerate for  $IJ = 1\frac{3}{2}$ . In the sum rules for the  $1\frac{3}{2}$  pentaquark, Eq.(39), the contribution of chiral odd terms are small compared with the chiral even terms, which leads to the degeneracy of the positive and negative parity states.

## V. DISCUSSION

### A. $J^P = \frac{3}{2}^-$ states

We have found that positive and negative parity states of the pentaquark baryons with high spin may exist in low mass region, which is not the case for ordinary three-quark baryons. The widths of these states are sensitive to the mass positions relative to the threshold of  $S$ -wave meson-baryon decay. For the  $0\frac{3}{2}^-$  and the  $1\frac{3}{2}^-$  states, no  $S$ -wave channel of meson-baryon decay is open, if their masses are below  $NK^*$  and  $\Delta K$  threshold, respectively. In the present analysis, we obtained  $1.5 \sim 1.7$  GeV for the mass of the  $0\frac{3}{2}^-$

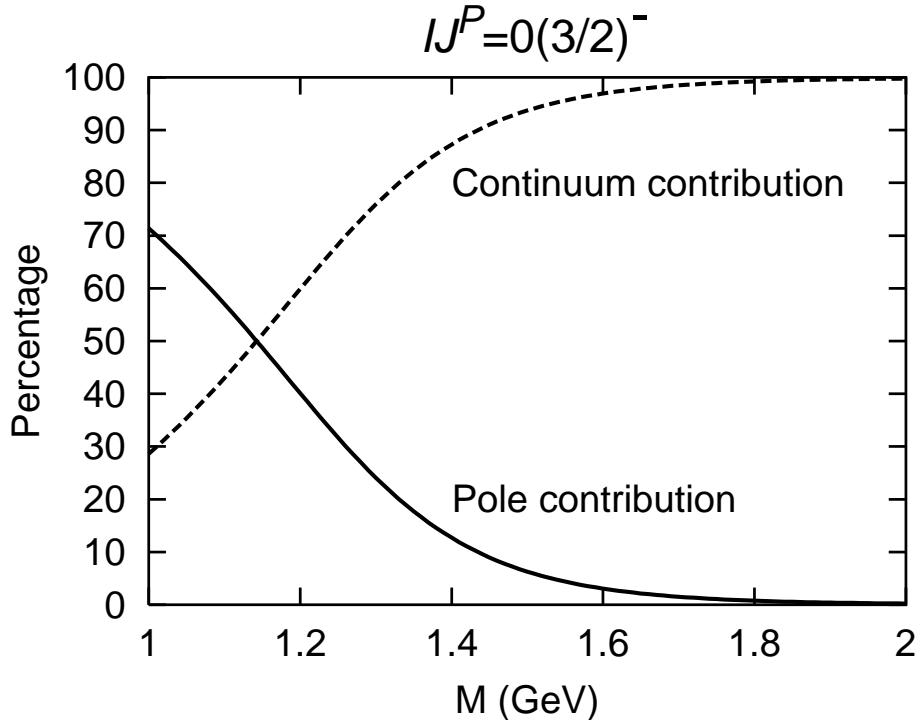


FIG. 4: Relative magnitude of the pole (solid line) and the continuum (dashed line) contribution in Eq.(35) for negative parity as functions of  $M$  with the effective continuum threshold  $\omega_-^{I=0} = 1.8 \text{ GeV}$ .

state, which is below the  $NK^*$  threshold energy. On the other hand, the predicted mass of the  $1\frac{3}{2}^-$  state,  $1.4 \sim 1.6 \text{ GeV}$ , is also below the  $\Delta K$  threshold. This means that both states can decay only to the  $D$ -wave  $NK$  states because of the conservation law of total spin and parity. Due to the high centrifugal barrier, the widths are strongly suppressed. As a result, these states may be observed as narrow peaks. The  $0\frac{3}{2}^-$  state can be a candidate of the observed  $\Theta^+(1540)$  and the  $1\frac{3}{2}^-$  pentaquark might be a new particle,  $\Theta^{++}$ .

The possibility of high spin states of the pentaquark has also been suggested from other approaches. Very recently, by employing a quark model with the meson exchange and one-gluon exchange interaction, negative parity  $uudd\bar{s}$  pentaquarks have been investigated [21]. The low lying states found in this calculation are  $0\frac{1}{2}^-$  and  $0\frac{3}{2}^-$  states. The former may be broad and not be observed since it lies above the  $NK$  threshold. On the other hand, the latter is below the  $NK^*$  threshold and therefore it can be observed as a narrow peak.

The existence of  $1\frac{3}{2}^-$  pentaquark as a low lying state has been suggested in Ref.[19]. In Ref.[19], a simple quark model in which constituent quarks interact via one-gluon exchange

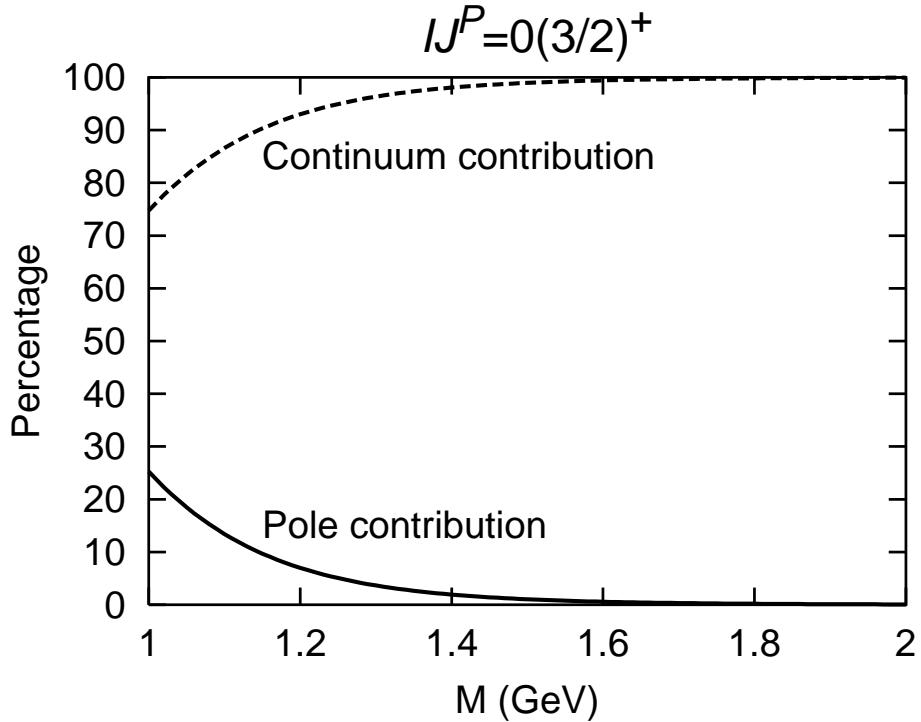


FIG. 5: Relative magnitude of the pole (solid line) and the continuum (dashed line) contribution in Eq.(35) for positive parity as functions of  $M$  with the effective continuum threshold  $\omega_+^{I=0} = 1.8 \text{ GeV}$ .

force at short distances and confining (or string) potential at long distances was considered. A  $qqqq\bar{q}$  system has a connected string configuration corresponding to a confined state, in addition to an ordinary meson-baryon like configuration. A variational method called AMD (antisymmetrized molecular dynamics) [30, 31] was applied to the confined five-body system with  $uudd\bar{s}$  and all the possible spin parity states with the connected string configurations were calculated. The narrow and low lying states they have found are  $0\frac{1}{2}^+$ ,  $0\frac{3}{2}^+$  and  $1\frac{3}{2}^-$  states. The former two states have just the same structure as that conjectured by Jaffe and Wilczek [11]. We represent it as  $[ud]_{S=0,I=0}[ud]_{S=0,I=0}[\bar{s}]$ , where  $[ud]_{S,I}$  denotes a color  $\bar{\mathbf{3}}$   $ud$ -diquark with spin  $S$  and isospin  $I$ . Both of the two diquarks gain color magnetic interaction since they have  $S = 0$ . However, the  $0\frac{1}{2}^+$  and  $0\frac{3}{2}^+$  states lose kinetic and string energy since the two diquarks, which are to be antisymmetric in color, are identical and since they must be relatively  $P$ -wave. In Ref.[19], another energetically favorable state has been predicted, which consists of an  $S = 0$  diquark and an  $S = 1$  diquark:  $[ud]_{S=0,I=0}[ud]_{S=1,I=1}[\bar{s}]$ . The quantum number of this state is totally  $1\frac{3}{2}^-$ . It loses color magnetic interaction due to the

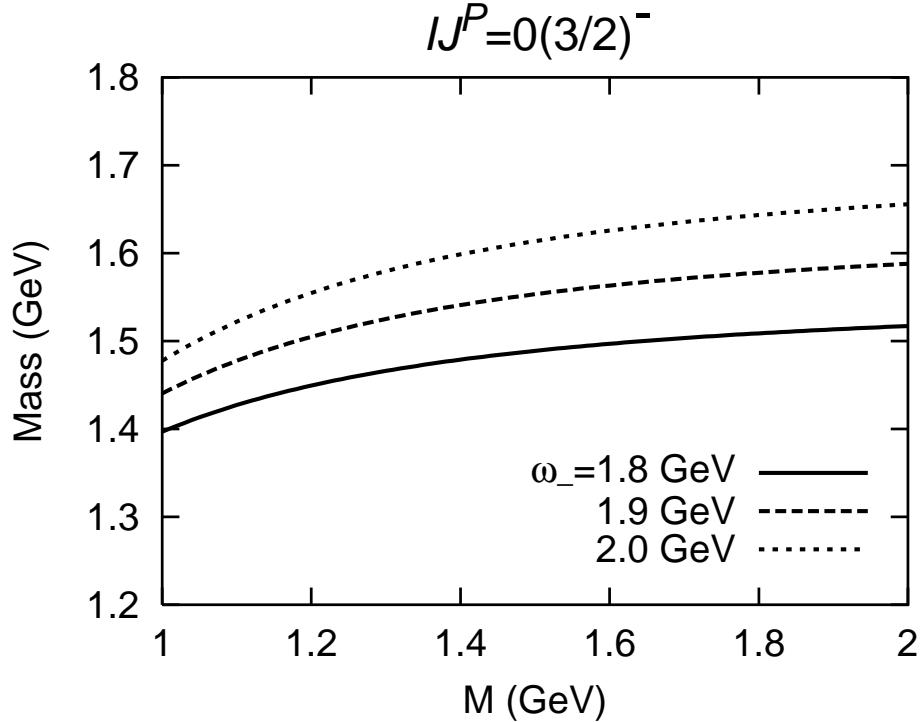


FIG. 6: Mass of the  $IJ^P = 0\frac{3}{2}^-$  pentaquark as a function of  $M$  with the effective continuum threshold  $\omega_-^{I=0} = 1.8$  GeV (solid line), 1.9 GeV (long-dashed), 2.0 GeV (short-dashed).

existence of a diquark with  $S = 1$ . However, the  $1\frac{3}{2}^-$  state gains kinetic and string energy since the two diquarks are no longer identical and since they can be relatively  $S$ -wave. Owing to the balance between the energy gain and the loss, the  $1\frac{3}{2}^-$  state degenerate with the  $0\frac{1}{2}^+$  and  $0\frac{3}{2}^+$  states.

A problem common to the above two works is that within the quark models one cannot predict the absolute masses but only the level structure of the pentaquarks in principle. The quark models employed in Refs.[19, 21] relies on the zero-point energy of the confining potential. The value of the zero-point energy, however, is not determined within this kind of empirical models. In Ref.[19], it was adjusted to reproduce the observed mass of  $\Theta^+$ . Whereas, the QCD sum rule is able to estimate the absolute mass though it depends on the effective continuum threshold. We confirmed from the QCD sum rule that the  $0\frac{3}{2}^-$  and the  $1\frac{3}{2}^-$  states are below the  $S$ -wave threshold and therefore they can be narrow states.

The pentaquark with  $1\frac{3}{2}^-$  has also been found to exist as a resonant state in the  $\Delta K$  channel [32] from the chiral unitary approach. This state is generated due to an attractive interaction in that channel existing in the lowest order chiral Lagrangian. The attractive

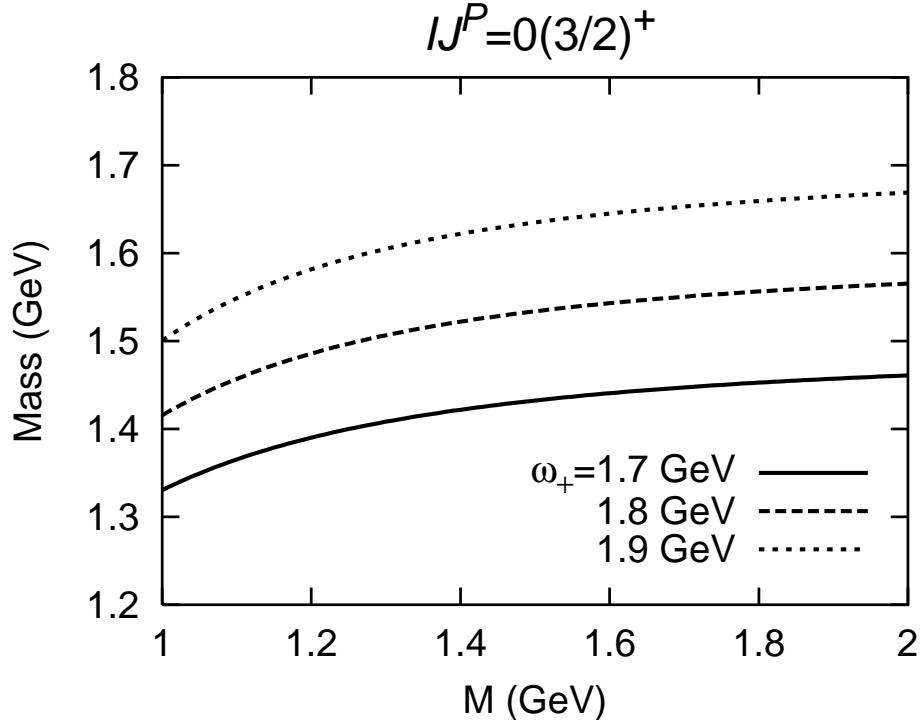


FIG. 7: Mass of  $IJ^P = 0\frac{3}{2}^+$  pentaquark as a function of  $M$  with the effective continuum threshold  $\omega_+^{I=0} = 1.7$  GeV (solid line), 1.8 GeV (long-dashed), 1.9 GeV (short-dashed).

interaction leads to a pole of the complex energy plane and manifests itself in a large strength of the  $\Delta K$  scattering amplitude with  $L = 0$  and  $I = 1$ . We note that the interpolating field, Eq.(21), can also couple with such a  $\Delta K$  resonance states because it contains the  $\Delta K$  component as is shown by Fierz transformation.

### B. $J^P = \frac{3}{2}^+$ states

The  $0\frac{3}{2}^+$  state has been discussed as a  $LS$  partner of the  $0\frac{1}{2}^+$  state [22]. In the QCD sum rule study of the  $0\frac{1}{2}^+$  state [18], it was found in the energy region compatible with the experimentally measured  $\Theta^+$  mass. In Ref.[18], the interpolating field based on Jaffe and Wilczek's conjecture:  $[ud]_{S=0,I=0}[ud]_{S=0,I=0}[\bar{s}]$  for the  $0\frac{1}{2}^+$  state was employed. If such a diquark structure is realized, the mass splitting between the  $0\frac{1}{2}^+$  and the  $0\frac{3}{2}^+$  is expected to be small because the effect of the spin-orbit force should be small due to the existence of two spinless diquarks. In Ref.[22], the authors predicted that the  $0\frac{3}{2}^+$  state may exist in the mass region  $1.54\text{GeV} \sim 1.68$  GeV based on the diquark picture. In the present calculation,

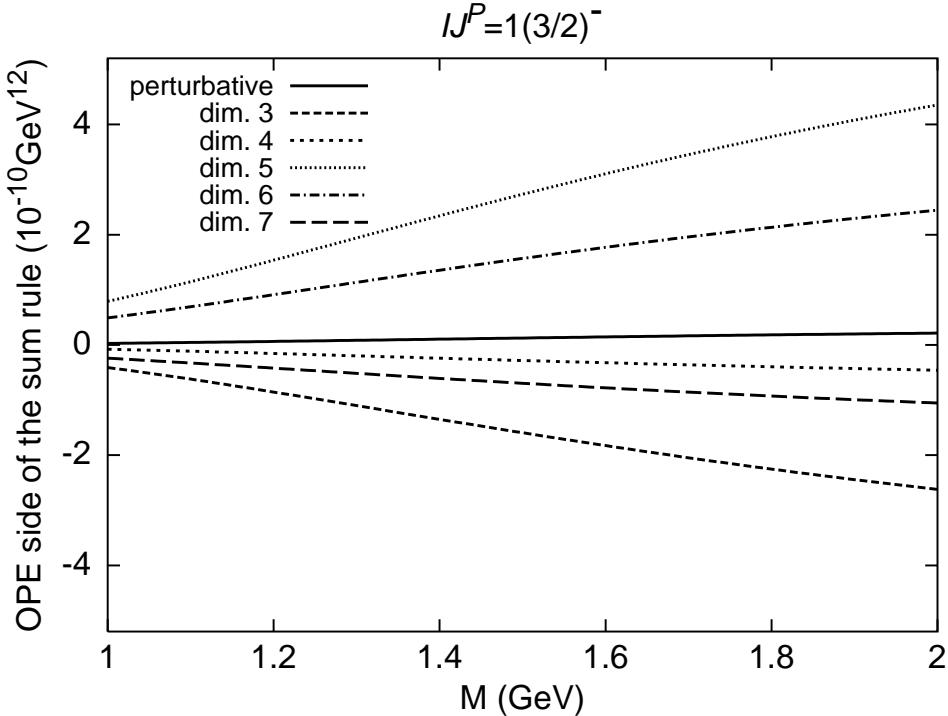


FIG. 8: Contributions of the terms in OPE side (the right hand side) of Eq.(39) for negative parity as functions of  $M$  with the effective continuum threshold  $\omega_-^{I=1} = 1.8$  GeV.

since the interpolating field for the  $I = 0$  can couple with such the diquark configuration, the obtained result for the  $0\frac{3}{2}^+$  state may be associated with the  $LS$  partner of the  $0\frac{1}{2}^+$  state with Jaffe and Wilczek's diquark structure. Our result implies a possibility of the  $0\frac{3}{2}^+$  state. However, as was shown in the previous section, the sum rule for  $0\frac{3}{2}^+$  channel is less reliable. Therefore we should not discuss the mass difference with the  $0\frac{1}{2}^+$  state using the results from the present sum rule. To do that, it would be necessary to construct the sum rule in which the background continuum contribution is made as small as possible.

The  $0\frac{3}{2}^+$  and  $1\frac{3}{2}^+$  pentaquarks are expected to be broader than the  $0\frac{3}{2}^-$  and  $1\frac{3}{2}^-$  states. The reason is as follows. The  $0\frac{3}{2}^+$  and  $1\frac{3}{2}^+$  states can decay into  $P$ -wave  $NK$  states, while the  $0\frac{3}{2}^-$  and  $1\frac{3}{2}^-$  states decay only to  $D$ -wave  $NK$  states. The centrifugal barrier of  $P$ -wave  $NK$  states is lower than that of  $D$ -wave  $NK$  states, which makes the positive parity states broader than the negative parity states. The present result for the  $1\frac{3}{2}^+$  state is consistent with a recent calculation by Skyrme model [33]. The authors in Ref.[33] predicted that there exists a new isotriplet of  $\Theta$ -baryons with  $1\frac{3}{2}^+$ . Its mass is 1595 MeV and the width is large:  $\Gamma \sim 80$  MeV.

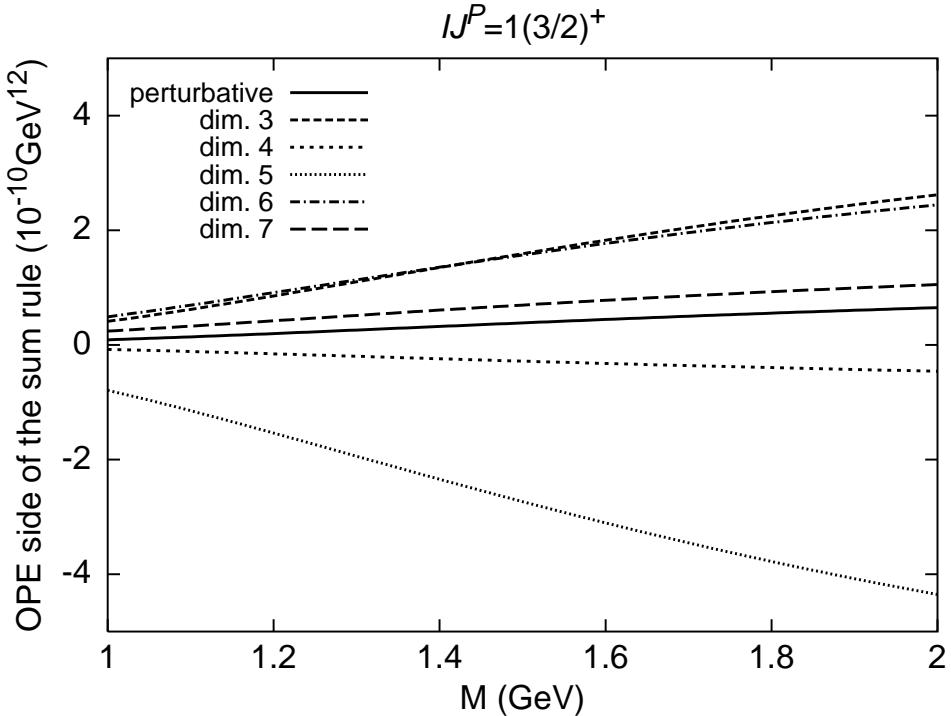


FIG. 9: Contributions of the terms in OPE side (the right hand side) of Eq.(39) for positive parity as functions of  $M$  with the effective continuum threshold  $\omega_+^{I=1} = 1.8$  GeV.

## VI. SUMMARY

In summary, we have studied the high spin ( $J = 3/2$ ) states of pentaquark with  $I = 0$  and  $I = 1$  using the method of QCD sum rule. We have derived the QCD sum rules for both of the negative and positive parity states. The QCD sum rule suggests the existence of pentaquark states with narrow width,  $IJ^P = 0\frac{3}{2}^-$  and  $1\frac{3}{2}^-$ . The masses for the  $I = 0$  and  $I = 1$  states are predicted to be  $1.5 \sim 1.7$  GeV and  $1.4 \sim 1.6$  GeV, which are much below the  $NK^*$  threshold and the  $\Delta K$  threshold, respectively. Since only the  $D$ -wave decay to  $NK$  channel is allowed, they should be extremely narrow states. Concerning the mass difference between  $IJ^P = 0\frac{3}{2}^-$  and  $1\frac{3}{2}^-$  states, we cannot say anything definitely, because the masses depend on the values of the effective continuum threshold in the present calculation. The QCD sum rule also shows the possibility of the existence of the  $J^P = 3/2^+$  states. The positive parity states may be broader than the negative parity states since they are allowed to decay into  $P$ -wave  $NK$  state.

It is worth mentioning that this is the first QCD sum rule analysis of high spin ( $J = 3/2$ )

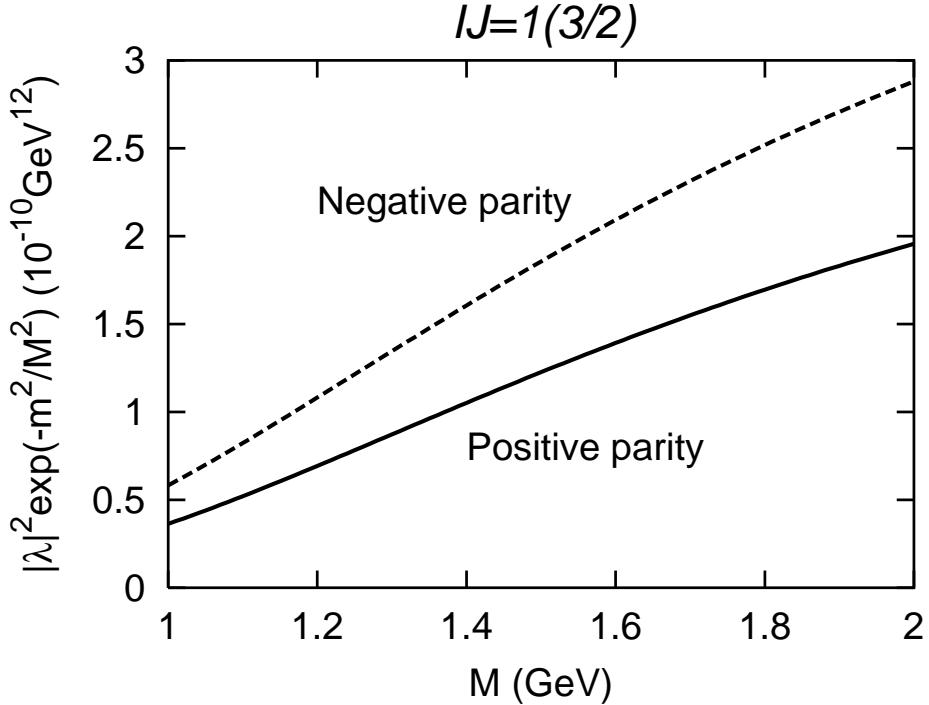


FIG. 10:  $|\lambda_{\pm}|^2 \exp(-\frac{m_{\pm}^2}{M^2})$  for  $I = 1$  as functions of  $M$  obtained from Eq.(35) with the effective continuum threshold  $\omega_{\pm}^{I=1} = 1.8$  GeV.

states of the pentaquark. The important point is that this work suggests possible existence of the high spin states in the same energy region as the  $J = 1/2$  states obtained by QCD sum rule in Ref.[18]. This abnormal spectra of the pentaquark are contrast to the ordinary baryon spectra. It is also remarkable that the exotic spin and parity,  $\frac{3}{2}^-$ , leads to the existence of extremely narrow states. It would be interesting to see if lattice calculation could confirm our findings since most of the existing works using QCD sum rules or lattice QCD have concentrated on  $J = \frac{1}{2}$  pentaquark states.

Finally, we would like to give a comment on the present formalism of QCD sum rules, which has been widely used for pentaquark. The formalism is a simple extension of that for the ordinary hadrons. One of the subtle problems in QCD sum rules for pentaquark is how to properly extract a resonance in the contamination of the background continuum states. Before we obtain a final conclusion for the pentaquark study with the QCD sum rule, it is a necessary process to examine the validity of the QCD sum rule formalism for exotic hadrons. In order to settle this problem, further experimental and theoretical studies on excited states and other pentaquark states with their spin and parity are desired. It is also

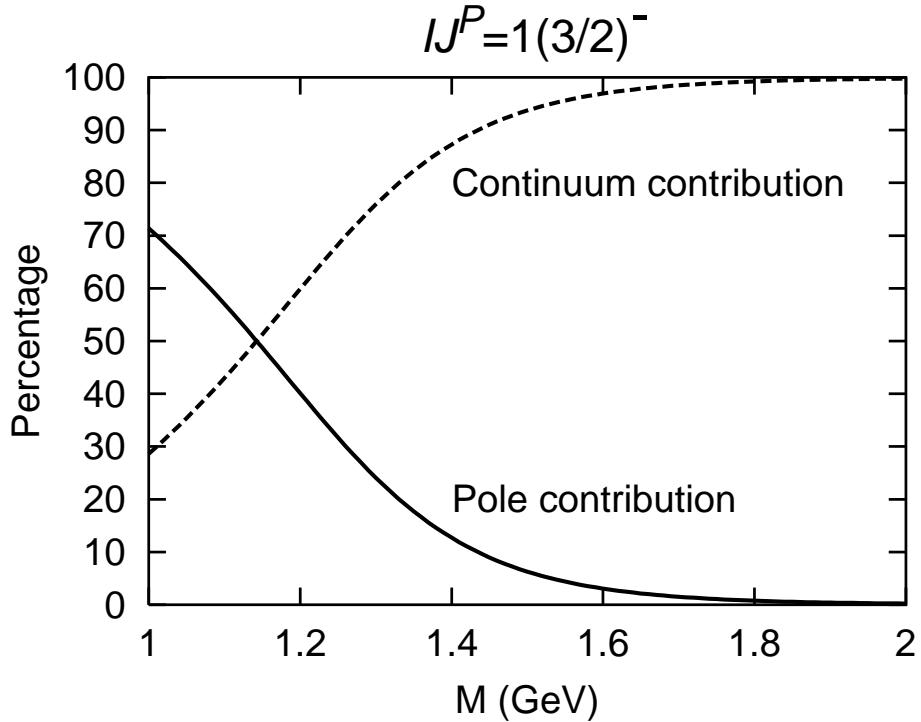


FIG. 11: Relative magnitude of the pole (solid line) and the continuum (dashed line) contribution in Eq.(39) for negative parity as functions of  $M$  with the effective continuum threshold  $\omega_-^{I=1} = 1.8 \text{ GeV}$ .

useful to compare the lattice QCD calculations with the QCD sum rule results.

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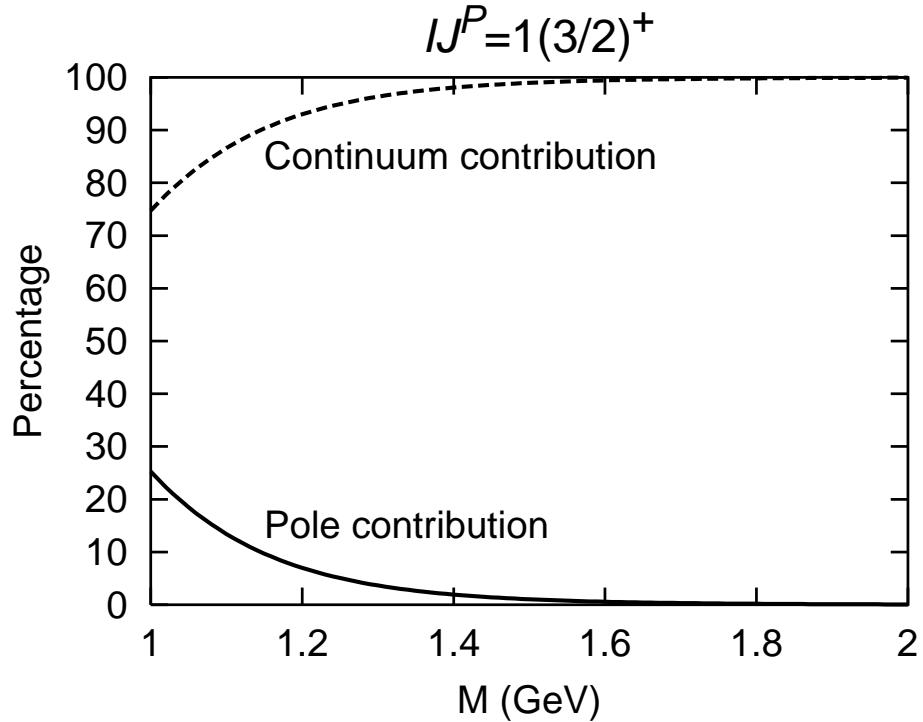


FIG. 12: Relative magnitude of the pole (solid line) and the continuum (dashed line) contribution in Eq.(39) for positive parity as functions of  $M$  with the effective continuum threshold  $\omega_+^{I=1} = 1.8 \text{ GeV}$ .

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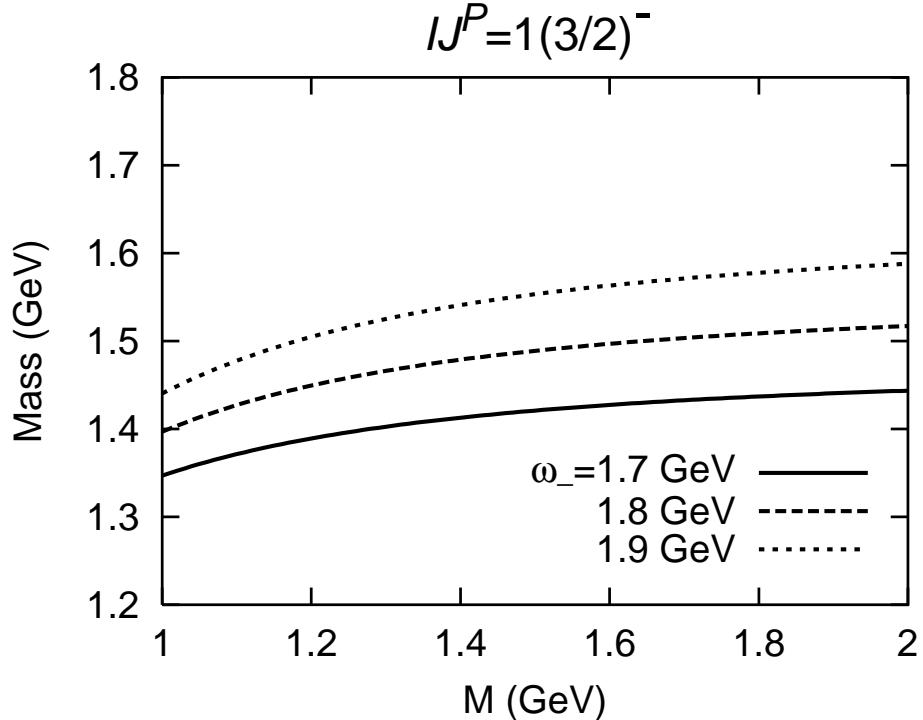


FIG. 13: Mass of  $IJ^P = 1\frac{3}{2}^-$  pentaquark as a function of  $M$  with the effective continuum threshold  $\omega_-^{I=1} = 1.7$  GeV (solid line), 1.8 GeV (long-dashed), 1.9 GeV (short-dashed).

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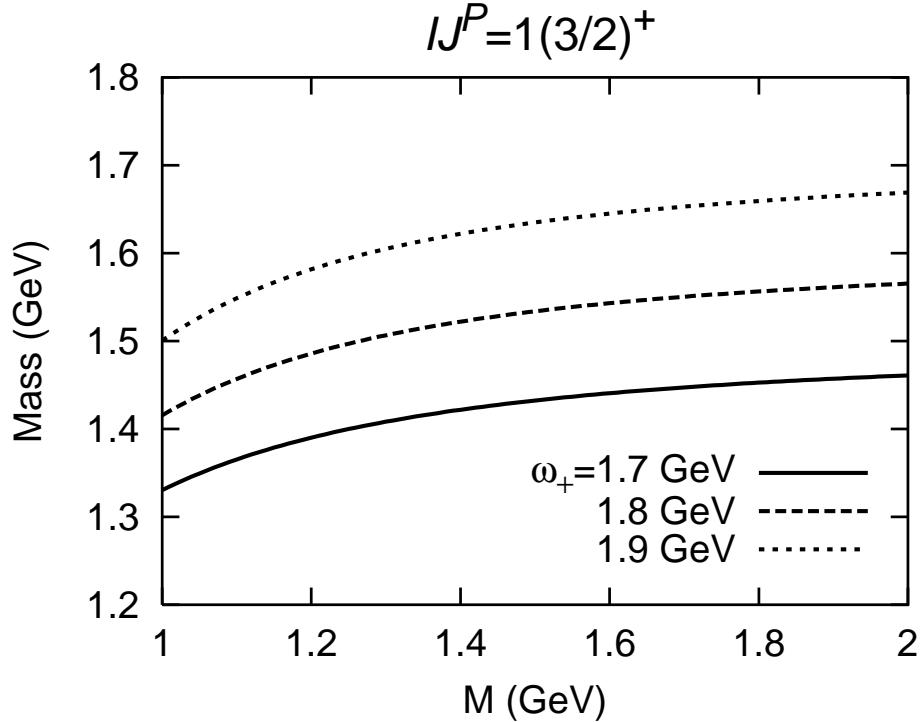


FIG. 14: Mass of  $IJ^P = 1\frac{3}{2}^+$  pentaquark as a function of  $M$  with the effective continuum threshold  $\omega_+^{I=1} = 1.7$  GeV (solid line), 1.8 GeV (long-dashed), 1.9 GeV (short-dashed).

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